Name_____________________________________ Name ____________________________________

Definition: Assume that *f(x)* exists for all x in some open interval containing a, except possible at a itself. We say that the limit of *f(x)* as x approaches a is L, written $\lim_{x \to a} f(x) = L$ if for *__any__* number $\varepsilon > 0$ there is a corresponding δ >0 such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$. NOTE We usually are given an episilon ε and we have to find the delta, δ for which the statement is true.

Another way of putting it is: We are given the possible tolerance (on either side) of the limit L and we have to find the allowable change in x that will keep the values for x within that window of tolerance.

Go to http://www.calculusapplets.com/formallimits.html

Notice at the top of the applet, there is a drop down menu, the first graph y = 0.5x, this is a nice polynomial so the limit is easy to find. (just plug and chug). If ε= .2 then move the sliders at the bottom of the page to see what the δ has to be so that the line comes out of the sides (or the corners) of the yellow box, but not the top or bottom.

Your value for δ = _______. This means, if _________< x < ________, then the distance

1-δ 1+ δ *

between the limit which is .5 and value of f(x) will be less than .2.

*[Note It is 1-δ and 1+δ because the "c" value is 1. In general the bounds for x are c -δ and c + δ].

Try a few values for C, keeping f(x) = 0.5 x.

C	f(C)	f(x)-L where L is the limit at C	ε	C -δ	C +δ

Rem: You don't have to get the exact δ that makes the graph of the function (black line) go through the corners, just ANY δ that keeps the graph out of the green areas!

First

So if c = ______ so the limit for f(c) is _________ , and my found value for δ is __________,

I can say if _______< x < _______, then the distance between f(x) and ______ is less than _______

1-δ 1+ δ L ε

Second

So if c = ______ so the limit for f(c) is _________ , and my found value for δ is __________,

I can say if _______< x < _______, then the distance between f(x) and ______ is less than _______

1-δ 1+ δ L ε

Go to the drop down menu again and try the second example from the drop down menu. The value c = 1, but f(1) =1.5. We would say that we have to lift our pencil to draw the graph, thus informally the function is not continuous. Given

ε = .2, can you find a δ such that every point (of the graph) is inside the yellow box?

Your δ =________. We say this function ________________________ at x = 1.

Continuous or discontinuous

Look at the other examples from the drop down menu. Play with the sliders to find a delta δ so that the graph does not go into the green areas.

Example 3: A line with a missing point c = 1 and ε = .1, then what is the δ such that

$\text{if } 0<|x-1|<\delta, \text{ then } |f(x)-0.5|<0.1$ Notice you have to change the sliders.

δ = _______

Rem: You don't have to get the exact δ that makes the graph of the function (black line) go through the corners, just ANY δ that keeps the graph out of the green areas!

Example 4: $y=\dfrac{\sin x}{x}$ c = 0 and ε = .1, then what is the δ such that

$\text{if } 0<|x-0|<\delta, \text{ then } \left|\dfrac{\sin x}{x}-1\right|<0.1$ δ = ______________

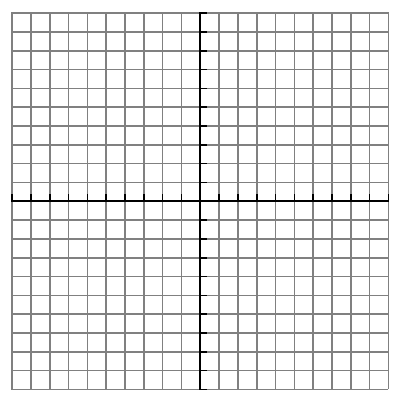
Look at the last few examples, do you see when there is no δ such that

$\text{if } 0<|x-a|<\delta, \text{ then } |f(x)-L|<\varepsilon.$ is true then the function is not continuous.

Try 2 functions of your own. Type the function in the box following *f(x)* under the graph, determine the limit L and choose a value to approach and a, epsilon ε >0. What would your δ be in order to keep the graph out of the green (coming out the sides of the yellow box and not out of the top or bottom). Include the box on your graphs.

Function f(x) = _________________________

c = _________L =________ ε = ___________



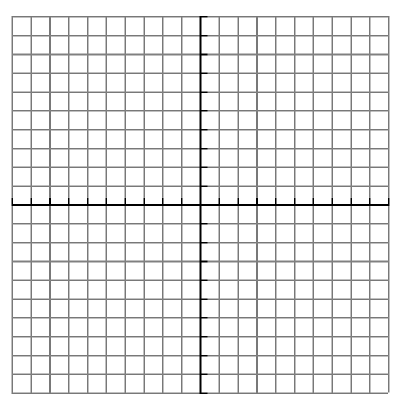
We find the δ to be ___________

Sketch the graph.

Find δ algebraically:

Function f(x) = _________________________

c = _________L =________ ε = ___________



We find the δ to be ___________

Sketch the graph.

Find δ algebraically:

Function f(x) = ___________________________

c = __________L =_________ ε = ____________

We find the δ to be ____________

Sketch the graph.

Find δ algebraically: